

# The NIMS 2008 Conference & The 4th East Asia SIAM Conference

October 10-12 2008  
National Institute for Mathematical Sciences (NIMS)  
Hotel Riviera, Daejeon, Korea

## October 10, Friday Morning

08:00 – 08:50	Registration	
08:50 – 09:00	Opening Ceremony: Hyung-Chun Lee & Tao Tang	
	Chair : Dongwoo Sheen	
09:00 – 09:50	SIAM Keynote Speakers I: <b>Douglas Arnold</b>	
09:50 – 10:00	Break	
	Session I (Chair: Abd Rahni Mt Piah)	Session II (Chair : Shinichi Oishi)
10:00 – 10:40	<b>Ya Yan Lu</b> Computing Dirichlet-to-Neumann Maps for Numerical Simulation of Photonic Crystal Structures	<b>Victor Didenko</b> Fredholm Properties of Operators Arising in Spline-Approximation Methods for Integral Equations with Conjugation
10:40 – 11:20	<b>Shao-Liang Zhang</b> Numerical Algorithms for Solving Shifted Complex Symmetric Linear Systems	<b>Hai-wei Sun</b> A High Order Compact Scheme for the Problem from Option Pricing with Jump
11:20 – 12:00	<b>M. Omar</b> A Production and Repair Model with Time-Varying Demand	<b>Xiao-Qing Jin</b> A Family of Generating Functions with Its Application in Pricing Options
12:00 – 14:00	Lunch	

**October 10, Friday Afternoon**

	Session I (Chair: Shao-Liang Zhang)	Session II (Chair: Xiao-Qing Jin)
14:00 – 14:25	<b>Takeshi Ogita</b> High Precision and Efficient Computation of Sparse Matrix-Vector Product	<b>Hyoseop Lee</b> Laplace transform method for the Black-Scholes equation
14:25– 14:50	<b>Katsuhisa Ozaki</b> A Robust Algorithm for Geometric Predicate by Sum of Determinants	<b>Sheon-Young Kang</b> An Integral equation method for the inverse problems of string vibration
14:50 – 15:15	<b>Tan Wang</b> On Decrease of Condition Number of Coefficient Matrices in the Cyclic Reduction for Linear Systems	<b>Sungkwon Kang</b> An inverse problem for the Helmholtz equation
15:15 – 15:30	Break	
	EASIAM Student Paper Competition (Chair: Tao Tang)	
15:30 – 15:50	<b>Takafumi Miyata</b> An Efficient Parallelization of the QR Algorithm for the Symmetric Tridiagonal Eigenproblems	
15:50 – 16:10	<b>Naoya Yamanaka</b> Fast Verified Automatic Integration Algorithm Over Finite Interval	
16:10 – 16:30	<b>Ho-Seok Lee</b> A first-passage-time model under regime-switching market environment	
16:30 – 16:50	<b>Ying-Ying Zhang</b> A Family of Generating Functions with An Application in Finance	
16:50 – 17:10	<b>Byung Hwa Lim</b> Optimal Investment, Consumption and Retirement Decision with Disutility and Liquidity Constraints	
17:10 – 17:20	Break	
	EASIAM Student Paper Competition (Chair: Dongwoo Sheen)	
17:20 – 17:40	<b>Zuliang Lu</b> A Posteriori Error Analysis of Triangular Mixed Finite Element Methods for Semilinear Quadratic Optimal Control Problems	
17:40 – 18:00	<b>Takehiko Kinoshita</b> On the $L^2$ a Priori Error Estimates to the Finite Element Solution of Elliptic Problems with Singular Adjoint Operator	
18:00 – 18:10	Photo Time	
18:10 – 20:00	Banquet	

**October 11, Saturday Morning**

08:00 – 09:00	Registration	
Chair: Hyung-Chun Lee		
09:00 – 09:50	SIAM Keynote Speakers I : <b>Olof B. Widlund</b> Domain Decomposition Theory for Less Regular Subdomains	
09:50 – 10:00	Break	
	Session I (Chair: Ya Yan Lu)	Session II (Chair: Byeong-Chun Shin)
10:00 – 10:40	<b>Chang-Ock Lee</b> Conductivity Imaging based on Harmonic Algorithms for Magnetic Resonance Electrical Impedance Tomography (MREIT)	<b>Hisashi Okamoto</b> Particle Trajectories Around a Running Cylinder in Brinkman's Porous-media Flow
10:40 – 11:20	<b>Xue-Cheng Tai</b> Graph Cuts for the Multiphase Mumford-Shah model Using Piecewise Constant Level Set	<b>Ruo Li</b> An Anisotropic Refinement Algorithm Based on Harmonic Mapping
11:20 – 12:00	<b>Abd Rahni Mt Piah</b> Rational Generalised Ball Functions for Monotonic Interpolating Curves	<b>Mitsuhiro T. Nakao</b> On the Constructive Error Estimates in the Finite Element Methods with Applications to the Numerical Verification of Solutions for Nonlinear PDEs
12:00 – 13:30	EASIAM Meeting & Lunch	

**October 11, Saturday Afternoon**

14:00 – 18:00	Tour
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**October 12, Sunday Morning**

	Session I (Chair: Jeonghwan Choi)	
09:00 – 09:40	<b>Sangdong Kim</b> Lower-order preconditioning for spectral element discretization to elliptic problems	
09:40 – 10:20	<b>Dongwoo Sheen</b> $P_1$ -nonconforming quadrilateral finite element for the Stokes problem	
10:20 – 11:00	<b>Yongwimon Lenbury</b> Modelling Delay Mechanisms in Human Physiology and Biology	
11:00 – 11:10	Break	
	Session I (Chair: Yongwimon Lenbury)	Session II (Chair: Shao-Liang Zhang)
11:10 – 11:35	<b>Kalyan Das</b> Analysis of Nutrient-Plant-Herbivore Recycling Model With Time Delay	<b>Siming Huang</b> Solving the Linear Operator Equation with Prior Information
11:35 – 12:00	<b>Hidenori Yasuda</b> Preparedness of the Influenza in the Commuter Towns of Tokyo; Analysis of Model Cities and a Metaphor Model	<b>Christian Keil</b> Verified Linear Programming and Extensions
12:00 – 12:25	<b>Imbunm Kim</b> Ratio-Dependent Predator-Prey Model with Diffusion	<b>Shin'ichi Oishi</b> Numerical Verification of Optimum Point in Linear Programming
12:25 – 14:00	Lunch	

	Session I (Chair: Victor Didenko)	Session II (Chair: Hisashi Okamoto)
14:00 – 14:25	<b>Seung Yeal Ha</b> A Simple Proof of the Cucker-Smale Flocking Dynamics and Mean-field Limit	<b>Siegfried M. Rump</b> Rigorous Error Bounds for Floating-Point Operations without Changing Rounding Mode
14:25– 14:50	<b>Janpou Nee</b> The Uniqueness and Existence of Chern-Simon Equation	<b>Chien-Hong Cho</b> On the Finite Difference Approximation for the Semilinear Wave Equation
14:50 – 15:15	<b>Young-Sam Kwon</b> Initial-Boundary Value Problems for Conservation Laws with Source Terms and the Degasperis-Procesi Equation	<b>Huimin Jing</b> SWE Model Based Simulation to Analyze the Cause of Pohang Harbor Hazard
15:15 – 15:30	Break	
<b>October 10, Friday Afternoon continued</b>		
	Session I (Chair: Taeyoung Ha)	Session II (Chair: Sangdong Kim)
15:30 – 15:55	<b>Myoungnyoun Kim</b> Verified Computations of Bifurcating Solutions for 3-dimensional Rayleigh-Bénard Convection Problems	<b>Jae-Hong Pyo</b> New Singular Function Method for Domain Singularities
15:55– 16:20	<b>Chi-Ok Hwang</b> New Directions of Statistical Physics in the Age of Computational Science: Yang-Lee Zero Phase Transition Approach for 2-D Triangular Antiferromagnets	<b>Yuki Ueda</b> A Set of Variant Hermite Elements for Dirichlet Boundary Problems and its Applications
16:20 – 16:45	<b>Mohd Salmi Md Noorani</b> Counting Closed Orbits of Hyperbolic Diffeomorphisms via Mertens Theorem	
16:45	Closing Remarks	

Douglas Arnold  
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# Domain Decomposition Theory for Less Regular Subdomains

Olof B. Widlund

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In the theory for domain decomposition methods, it has previously often been assumed that each subdomain is the union of a small set of coarse shape-regular triangles or tetrahedra. Recent progress is reported, which makes it possible to analyze cases with irregular subdomains such as those produced by mesh partitioners. The goal is to extend the analytic tools so that they work for problems on subdomains that might not even be Lipschitz and to characterize the rates of convergence of domain decomposition methods in terms of a few, easy to understand, geometric parameters of the subregions. For two dimensions, some best possible results have already been obtained for scalar elliptic and compressible and almost incompressible linear elasticity problems; the subdomains should be John or Jones domains and the rates of convergence are determined by parameters that characterize such domains and that of an isoperimetric inequality.

# Computing Dirichlet-to-Neumann Maps for Numerical Simulation of Photonic Crystal Structures

Ya Yan Lu

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Photonic crystals (PhCs) have promising applications in future integrated circuits based on lightwaves. A typical PhC structure or device has many identical unit cells. Based on the Dirichlet-to-Neumann maps of the unit cells, we have developed efficient numerical methods for analyzing various PhC structures. In this paper, these works are briefly reviewed.

# Numerical Algorithms for Solving Shifted Complex Symmetric Linear Systems

Shao-Liang Zhang

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In this talk, we consider the solution of shifted complex symmetric linear systems. Such systems arise in large-scale electronic structure simulations and there is a strong need for the fast solution of the systems. With the aim of solving the systems efficiently, we consider a special case of the QMR method for non-Hermitian shifted linear systems and propose its weighted quasi-minimal residual approach. A numerical algorithm, referred to as shifted QMR SYM(B), is given by the choice of a particularly cost-effective weight. Numerical examples are presented to show the performance of the shifted QMR SYM(B) method.

This is a joint work with Dr. Tomohiro Sogabe.

# A Production and Repair Model with Time-Varying Demand

M. Omar

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In this paper, we consider a production system that satisfies a continuous time-varying demand for a finished product over a known and finite planning horizon by supplying either new products or repaired used products. We assume that new products are fabricated from a single type of raw material procured from external suppliers, while used products are collected from the customers and then repaired to an as new condition before being sold again. The problem is to determine a joint policy for raw materials procurement, new products fabrication and used products repair such that the total relevant cost of the system is minimized. We propose a numerical solution procedure using a Microsoft Excels Solver tool. Then, the procedure is illustrated with a numerical example.

This is a joint work with Ivan Yeo.

# Fredholm Properties of Operators Arising in Spline-Approximation Methods for Integral Equations with Conjugation

Victor Didenko

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Let  $I$  denote the interval  $[0, 1]$ , and let  $a, b, c, d : I \mapsto \mathbb{C}$  and  $h_1, h_2 : I \times I \mapsto \mathbb{C}$  be continuous functions. Integral equations of the type

$$(A\varphi)(t) = a(t)\varphi(t) + \frac{b(t)}{\pi i} \int_I \frac{\varphi(s)}{s-t} ds + c(t)\overline{\varphi(t)} + \frac{d(t)}{\pi i} \int_I \frac{\overline{\varphi(s)}}{s-t} ds \\ + \int_I k_1(t, s)\varphi(s)ds + \int_I k_2(t, s)\overline{\varphi(s)}ds = f(t), \quad t \in I.$$

where  $\varphi$  is an unknown function, often arise in hydrodynamics, elasticity theory, acoustics, and various projection methods are a very popular approach for their solution. The applicability of such algorithms depends on the invertibility of some associated operators  $A_\tau, \tau \in I$ . In case of spline-approximation methods, these operators  $A_\tau$  belong to well-known operator algebras, and for  $\tau \in (0, 1)$  one can obtain simple conditions of their invertibility. In contrast, the operators  $A_0, A_1$  have much more complicated structure and, as far as we know, there is no effective tool to check their invertibility. Therefore, here we deal with a slightly weaker problem, viz. we study Fredholm properties of the operators  $A_0$  and  $A_1$ . In particular, it is proved that the indices of these operators belong to the set  $-3, -2, -1, 0, 1, 2, 3$ . On the other hand, we were not able to find any operator  $A_0$  with the index 3 or  $-3$ . Thus a more detailed study of this problem is needed.

# A High Order Compact Scheme for the Problem from Option Pricing with Jump

Hai-wei Sun

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In this paper we consider a high order compact scheme for the option pricing in jump-diffusion model. The value of a contingent claim under a jump-diffusion process satisfies a partial integro-differential equation (PIDE). We propose a fourth order compact finite difference scheme for discretizing the spatial variable of this PIDE. Then we employ the Simpson rule to evaluate the jump integral term, which results a Toeplitz-like matrix. At each time step, the computation involves matrix-vector multiplication, where the matrix is Toeplitz-like. Hence the fast Fourier transform is used to perform these multiplications efficiently. Numerical results are given to show that our approach gives fourth order accuracy in space which is much better than the classical second order central difference scheme.

This is a joint work with Spike T. Lee (University of Macau, Macao, China).

# A Family of Generating Functions with Its Application in Pricing Options

Xiao-Qing Jin

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We introduce the definition of a family of generating functions (FGF) of Toeplitz matrices, which is a generalization of the generating function of Toeplitz matrix. The FGF has an important application in pricing derivatives and provides a rigorous theoretical basis for the convergence analysis of the conjugate gradient method. All aspects related to the generating function could be extended to the FGF.

This is a joint work with Y. Y. Zhang and S. W. Vong. The research was partially supported by the research grant RG-UL/07-08S/Y1/JXQ/FST from University of Macau.

# Conductivity Imaging based on Harmonic Algorithms for Magnetic Resonance Electrical Impedance Tomography (MREIT)

Chang-Ock Lee  
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Magnetic Resonance Electrical Impedance Tomography (MREIT) is a late medical imaging modality visualizing static conductivity images of electrically conducting subjects. When we inject current into the object, it produces internal distributions of current density  $\mathbf{J}$  and magnetic flux density  $\mathbf{B} = (B_x, B_y, B_z)$ . By using an MRI scanner, we can measure  $B_z$  data where  $z$  is the direction of the main magnetic field of the scanner. Conductivity images are reconstructed based on the relation between the injection current and  $B_z$  data. Recently, MREIT has rapidly progressed in its theory, algorithm, and experiment technique and now reached to the stage of *in vivo* animal experiments. In this talk, we present the basic concept of MREIT, a recent MREIT algorithm called *local harmonic  $B_z$ -algorithm*, and a software named CoReHA. Furthermore, we discuss problems in the area of scientific computation, image processing, and mathematical modeling, which arise in MREIT.

This is a joint work with Kiwan Jeon, Eung Je Woo, Jin Keun Seo.

# Graph Cuts for the Multiphase Mumford-Shah model Using Piecewise Constant Level Set

Xue-Cheng Tai

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The piecewise constant level set method has previously been successfully used for the multiphase Mumford-Shah model. The resulting minimization problem can be solved by continuous optimization techniques such as the augmented lagrangian method. In this work, we instead propose an integer optimization technique to solve the multiphase Mumford-Shah functional represented by piecewise constant level set functions. This approach, which is based on a cut on an appropriate graph, is very superior in terms of efficiency compared to the previous methods. Numerical experiments show that the new method produces the same quality of results.

This talk is based on a joint work with Egil Bae.

# Rational Generalised Ball Functions for Monotonic Interpolating Curves

Abd Rahni Mt Piah

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A curve interpolating scheme is developed using rational generalised Ball functions with quartic numerator and linear denominator. The scheme has a unique representation, with parameters that can be used either to change the shape of the curve or to increase its smoothness. Conditions are derived for preserving monotonicity, and for achieving either  $C^1$  or  $C^2$  continuity. Some numerical results are presented.

This is a joint work with Samsul Ariffin Abdul Karim and Keith Unsworth.

# Particle Trajectories Around a Running Cylinder in Brinkman's Porous-media Flow

Hisashi Okamoto

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We study the movement of particles around a running cylinder. We suppose that an infinitely long circular cylinder is moving at a constant velocity. Then we compute the particle paths of the particles around the cylinder. J. C. Maxwell considered the problem in irrotational flow of inviscid fluid. He represented solutions by elliptic functions and showed that the particle paths form elastic curves. We consider here a similar problem in Brinkman's porous-media flow. In this case, our numerical examinations reveals some new interesting features of the particle trajectories, which are not observed in the case of irrotational flow.

This is a joint work with Mayumi Shōji, Department of Mathematical and Physical Sciences, Japan Women's University.

# An Anisotropic Refinement Algorithm Based on Harmonic Mapping

Ruo Li

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Motivated by the moving mesh method based on harmonic mapping, we propose a new anisotropic mesh refinement algorithm on unstructured meshes. Our algorithm uses scalar values as refinement indicators rather than matrices or vectors. The preferred refinement direction is determined by analyzing the element deformation based on the solution of the local harmonic mapping. Thus the algorithm does not depend on local recovery or the error estimate, as do many existing approaches mentioned in the references. This algorithm is implemented in both two and three dimensional spaces, and proven to be effective for the interpolation of functions with different types of anisotropic singularities and the solution of a 2nd order elliptic equation with discontinuity in its gradient. For smooth functions, the error of the numerical approximations can asymptotically achieve the optimal convergence order with respect to the number of degrees of freedom, illustrating that the error is equally distributed on all degrees of freedom. Especially, for discontinuous functions, the anisotropic convergence order is higher than the isotropic convergence order. In both function interpolation and equation solution cases, anisotropic refinement saves a big amount of computational cost.

# On the Constructive Error Estimates in the Finite Element Methods with Applications to the Numerical Verification of Solutions for Nonlinear PDEs

Mitsuhiro T. Nakao

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In this talk, we first consider the guaranteed a priori error estimates in the finite element method for Poisson's equation, bi-harmonic problems as well as Stokes problems. Next, as an application of the results, we show the numerical verification method of solutions for nonlinear elliptic problems and Navier-Stokes equations. The special emphasis of our method is that we can obtain the finite element solution with guaranteed error bounds even if we have no information about the existence of exact solutions for the original equations such as noncoercive or nonlinear problems. Several numerical examples which confirm the actual effectiveness of our method will be presented.

- [1] Nakao, M.T., Yamamoto, N. & Kimura, S., On best constant in the optimal error estimates for the  $H_0^1$ -projection into piecewise polynomial spaces, *Journal of Approximation Theory* 93 (1998), 491-500.
- [2] Kikuchi, F., Liu, Xuefeng, Determination of the Babuska-Aziz constant for the linear triangular finite element, *Japan Journal of Industrial and Applied Mathematics* 23 (2006), 75-82.
- [3] Nagatou, K., Hashimoto, K, Nakao, M.T., Numerical verification of stationary solutions for Navier-Stokes problems, *Journal of Computational and Applied Mathematics* 199 (2007), 424-431.
- [4] Hashimoto, K., Kobayashi, K., Nakao, M.T., Verified numerical computation of solutions for the stationary Navier-Stokes equation in nonconvex polygonal domains, *Hokkaido Mathematical Journal*, Vol. 36, Special Issue, Proceedings on "The First China-Japan-Korea Joint Conference on Numerical Mathematics" (2007), 777-799.
- [5] Nakao, M.T. and Hashimoto, K., Guaranteed error bounds for finite element approximations of noncoercive elliptic problems and their applications, *Journal of Computational and Applied Mathematics* 218 (2008), 106-115.

# Lower-order preconditioning for spectral element discretization to elliptic problems

Sangdong Kim

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The spectral element discretization to solve a second-order uniformly elliptic partial differential equation leads to a linear equation which needs efficient iterative methods such as Schwarzbased methods, preconditioning techniques related to multigrid methods. This is because such linear systems have large condition numbers usually dependent on the mesh sizes  $h$  and degrees  $N$  of polynomials adopted by spectral element methods. Hence it is very important to provide an optimal preconditioner in the sense that the preconditioned system has small condition numbers which are not dependent on  $h$  and  $N$ .

# $P_1$ -nonconforming quadrilateral finite element for the Stokes problem

Dongwoo Sheen

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with Chunjae Park (Konkuk University) and Byeong-Chun Shin (Chonam National University)

We propose a lowest-order stable nonconforming finite element pair for Stokes problems in two dimension, which requires the smallest degrees of freedom among all existing stable Stokes finite elements. The velocity approximation is based on  $P_1$ -quadrilateral decomposition of the domain. For each quadrilateral the velocity components will be approximated by piecewise linear polynomials which are continuous at midpoints of common edges of contiguous quadrilaterals while the pressure will be approximated by piecewise constant on each quadrilateral. In order to meet the Babuška-Brezzi inf-sup condition, the velocity components will be augmented by bubble functions, which can be eliminated at implementation level. Optimal order error estimates are derived. A numerical example is presented.

# Modelling Delay Mechanisms in Human Physiology and Biology

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Many types of delay mechanisms have been identified to operate in physiological and biological systems. In population dynamics, delays due to maturation time have long been recognized and modeling attempts made with some success. We will discuss some new results on nonlinear differential equation population model with constant time delay. On the other hand, in human physiology, delays due to late responses in hormone secretion positive (feed-forward) and (feedback) control loops in a cascade system may be investigated through bifurcation analysis leading to interesting clinical interpretation. For other systems, application of a novel theory, such as knots theory or nonlinear theory involving the omega limit set, may be necessary. A third type of delay action has been observed in late recovery time after interruption of treatments. We shall illustrate this type of delays by numerical simulations of a nonlinear model of a bio-system of interest. Such discoveries can have far reaching implications for disease diagnosis and control.

This is a joint work with D. Giang (Hanoi Institute of Mathematics), A. De Gaetano (CNR IASI Laboratorio di Biomatematica), P. Pornsawad and C. Rattanakul (Mahidol University).

# High Precision and Efficient Computation of Sparse Matrix-Vector Product

Takeshi Ogita

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Computing a sparse matrix-vector product is a ubiquitous task in scientific computing. On the other hand, the quality of results obtained by floating-point arithmetic sometimes becomes poor because of the rounding errors. The purpose of this research is to develop a fast and (arbitrarily) high-precision algorithm of computing a matrix-vector product  $y = Ax$  with  $A$  being a large sparse matrix and  $x$  a (dense) vector. The proposed algorithm is based on fast and efficient algorithm of calculating sum and dot product developed by the authors. Numerical results are presented.

This is a joint work with Siegfried M. Rump, Shin'ichi Oishi.

# A Robust Algorithm for Geometric Predicate by Sum of Determinants

Katsuhisa Ozaki

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This talk is concerned with the 2D orientation problem, which is one of the basic problems in computational geometry. The problem is to know which side the point is left or right, or on the oriented line. This problem can be boiled down to the determinant predicate, i.e. whether the sign of the determinant is positive, negative or zero.

To obtain a correct sign of determinant, several methods transform this determinant into sum of floating-point numbers without rounding errors. After this transformation, accurate summation algorithms can be applied and a correct result is obtained.

In this talk, we propose a different kind of an accurate algorithm. Three points are given for this problem. We split all points into sum of floating-point numbers. Using the linearity of a determinant in one of the column vectors, we can expand the determinant into sum of determinants without rounding errors. After obtaining this expansion, some determinants, which have relatively large elements in magnitude, can be computed without rounding errors. To give an a priori error analysis for this computation, we investigate a verified numerical algorithm for this predicate. Finally, we present numerical examples in order to confirm effectiveness of the proposed method.

This is a joint work with Takeshi Ogita (Tokyo Woman's Christian University), Shin'ichi Oishi (Waseda University).

# On Decrease of Condition Number of Coefficient Matrices in the Cyclic Reduction for Linear Systems

Tan Wang

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The cyclic reduction is a direct method for solving linear systems with tridiagonal coefficient matrix  $T$ . In the cyclic reduction, non-zero off-diagonal entries are away from the diagonal part, and  $T$  is finally transformed to a diagonal matrix. After first step of the cyclic reduction, coefficient matrix becomes a pentadiagonal matrix  $P$ . To the best of our knowledge, the relationship between  $T$  and  $P$  has not been reported from the viewpoint of matrix condition number. In this talk, it is proved that the condition number of  $P$  is smaller than that of  $T$  if  $T$  is a positive definite matrix without multiple eigenvalues. The key point is that  $P^{-1}$  can be just expressed by using  $T^{-1}$ . By taking into account that the eigenvalues of positive definite  $T$  are not multiple, it is shown that the condition number of  $P$  always is smaller than that of  $T$ .

This is a joint work with Masashi Iwasaki, Yoshimasa Nakamura.

# Laplace transform method for the Black-Scholes equation

Hyoseop Lee

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with Dongwoo Sheen (Seoul National University)

In this talk, we consider the numerical solution of Black-Scholes equation. Instead of using the traditional time-marching algorithms, such as backward Euler and Crank-Nicolson methods, we employ the Laplace transformation method based on numerical Laplace inversion formula, introduced earlier by Sheen, Sloan, and Thomeé in 2000 and 2003, on a suitably chosen contour. The method has been shown to be exponentially convergent and naturally parallelizable. Several options are priced numerically and the practical parallelization performance will be shown.

# An Integral equation method for the inverse problems of string vibration

Sheon-Young Kang  
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An inverse problem of string vibration is examined to reconstruct the distributed or concentrated forces exerted on the string when the deflection data are given. Instead of using a finite number of lumped points, the present inverse problem is formulated by a Fredholm integral equation of the first kind,

$$\int_0^l g(x, \xi)p(\xi)d\xi = y(x),$$

where the kernel  $g(x, \xi)$  is non-smooth along the main diagonal,  $x = \xi$ . A novel technique is introduced and applied to this type of integral equation. It adopts Chebyshev polynomial to approximate the whole integrand including the unknown distributed or concentrated forces. To check the numerical accuracy, three examples of string deflections are solved and validated.

# An inverse problem for the Helmholtz equation

Sungkwon Kang

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A computational algorithm for the inverse problem for the two-dimensional Helmholtz equation is considered. Based on the scattered acoustic or electromagnetic fields, the shape of the scatterer is reconstructed. The linear sampling method will be used for the inversion.

# An Efficient Parallelization of the QR Algorithm for the Symmetric Tridiagonal Eigenproblems

Takafumi Miyata, Yusaku Yamamoto, Shao-Liang Zhang  
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We propose a fully pipelined multishift QR algorithm to compute all the eigenvalues of a symmetric tridiagonal matrix on parallel machines. Existing approaches for parallelizing the tridiagonal QR algorithm, such as the conventional multishift QR algorithm and the deferred shift QR algorithm, have suffered from either inefficiency of processor utilization or deterioration of convergence properties. In contrast, our algorithm realizes both efficient processor utilization and improved convergence properties at the same time by adopting a new shifting strategy. Numerical experiments on a shared memory parallel machine (Fujitsu PrimePower HPC2500) with 32 processors show that our algorithm is up to 1.9 times faster than the conventional multishift algorithm and up to 1.7 times faster than the deferred shift algorithm.

# Fast Verified Automatic Integration Algorithm Over Finite Interval

Naoya Yamanaka

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Assume  $f(x) \in C^\infty([a, b])$ . We consider the following two types of integrals with respect to  $x$  from  $a$  to  $b$ :

$$I_1 = \int_a^b f(x) dx$$
$$I_2 = \int_a^b \frac{f(x)}{\sqrt{1-x^2}} dx.$$

To verify these types of integrals using numerical computations, the evaluations of rounding error and truncation error are needed.

Rounding error for verified computations is more or less calculated by interval arithmetic, but the method is much slower than pure floating-point arithmetic. Furthermore, to get the upper bound, whole calculation by interval arithmetic must be completed. To solve these problems, we stress that Kashiwagi's method, which is an a priori error algorithm to calculate the upper bound of rounding errors in floating-point arithmetic, is useful. After running some algorithm with Kashiwagi's method once, we only need to execute pure floating-point arithmetic, so that verified computation is expected to get much faster.

The evaluation of truncation error depends on the quadrature in use. In this talk we adopt double exponential formula, gauss-legendre quadrature and clenshaw-curtis quadrature, romberg quadrature for  $I_1$ , and double exponential formula and gauss-chebyshev quadrature for  $I_2$  respectively. Truncation error of these quadratures have the upper bound of  $|f(z)|$  on  $D$ , which  $D$  is a domain on complex plane and depends on each quadrature. On conventional verified algorithm the domain  $D$  is fixed. Though the algorithm using a fixed domain can solve some difficult problem, the execution time of that tends to slow because the number of points satisfying the tolerance is larger. In this talk, we propose the algorithm for estimation of truncation error that calculates  $|f(z)|$  on several domains adaptively. Using our algorithm, the number of points satisfying the tolerance is smaller than the calculation when on a fixed domain, so that we can expect our algorithm to be faster.

This is a joint work with Takeshi Ogita (Tokyo Woman's Christian University), Masahide Kashiwagi (Waseda University), Nobito Yamamoto (The University of Electro-Communications), Shin'ichi Oishi (Waseda University).

# A first-passage-time model under regime-switching market environment

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In this paper, we suggest a first-passage-time model which can explain default probability and default correlation dynamics under stochastic market environment. We add a Markov regime-switching market condition to the first-passage-time model of Zhou(2001). Using this model, we try to explain various relationship between default probability, default correlation, and market condition. We also suggest a valuation method for credit default swap (CDS)with(or without) counterparty default risk (CDR) and basket default swap under this model.

Our numerical results provide us with several meaningfulimplications. First, default swap spread is higher in economic recession than in economic expansion across default swap maturity. Second, as the difference of asset return volatility between under bear market and under bull market increases, CDS spread increases regardless of maturity. Third, the bigger the intensity shifting from bull market to bear market, the higher the spread for both CDS without CDR and basket default swap.

# A Family of Generating Functions with An Application in Finance

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We introduce the definition of a family of generating functions (FGF) of Toeplitz matrices, which is a generalization of the generating function of Toeplitz matrix. The FGF has an important application in pricing derivatives. The pricing of a European call option leads under certain assumptions to a partial integro-differential equation (PIDE) without a convection term. We then use the FGF to analyze the convergence rate of the preconditioned conjugate gradient (PCG) method with Strang's circulant preconditioner for solving Toeplitz systems arising from a discretization of the PIDE. We show that if the FGF has certain properties, then the spectrum of the preconditioned matrix is clustered around 1, and moreover the smallest eigenvalue of the preconditioned matrix is uniformly bounded away from 0. It follows that the convergence rate of the PCG method is superlinear. The FGF provides a rigorous theoretical basis for the convergence analysis of the conjugate gradient method. All aspects related to the generating function could be extended to the FGF.

This is a joint work with Seak-Weng Vong and Xiao-Qing Jin.

# Optimal Investment, Consumption and Retirement Decision with Disutility and Liquidity Constraints

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## Abstract

In this paper we consider general consumption, portfolio and retirement optimization problems in which a working investor has liquidity constraints. Closed-form solutions are obtained for the utility maximization problems and numerical procedures are given for the general utility function under the liquidity constraints. The numerical results for a special utility function, for example, the constant relative risk aversion (CRRA) utility function, suggest that the restriction to borrow future income makes the investor retire in a lower critical wealth level than in the case of no liquidity constraints.

*Keywords* : Liquidity constraints, general utility function, consumption, portfolio selection, retirement, disutility, labor income

## References

- [1] K.J. CHOI AND H.K. KOO, *A Preference Change and Discretionary Stopping in a Consumption and Portfolio Selection Problem*, Math. Meth. Oper. Res, 61 (2005), pp. 419-435.
- [2] K.J. CHOI AND G. SHIM, *Disutility, Optimal Retirement, and Portfolio Selection*, Math. Financ., 16 (2006), pp. 443-467.
- [3] K.J. CHOI, G. SHIM, AND Y.H. SHIN, *Optimal portfolio, Consumption-Leisure and Retirement Choice Problem with CES utility*, Math. Financ., to appear (2008)
- [4] J.C. COX AND C.F. HUANG, *Optimum Consumption and Portfolio Policies When Asset Prices Follow a Diffusion Process*, J. Econ. Theory, 49 (1989), pp. 33-83.
- [5] P.H. DYBVIK AND H. LIU, *Lifetime Consumption and Investment: Retirement and Constrained Borrowing*, Working Paper, University of Washington (2005)
- [6] E. FARHI AND S. PANAGEAS, *Saving and Investing for Early Retirement: A Theoretical Analysis*, J. Financ. Econ., 83 (2007), pp. 87-121.
- [7] H. HE AND H.F. PAGÈS, *Labor Income, Borrowing Constraints, and Equilibrium Asset Prices*, Econ. Theory, 3 (1993), pp. 663-696.
- [8] M. JEANBLANC, P. LAKNER, AND A. KADAM, *Optimal Bankruptcy and Consumption/Investment Policies on an Infinite Horizon with a Continuous Debt Repayment Until Bankruptcy*, Math. Oper. Res., 29 (2004), pp. 649-671.
- [9] I. KARATZAS, J.P. LEHOCZKY, S.P. SETHI, AND S.E. SHREVE, *Explicit Solution of a General Consumption/Investment Problem*, Math. Oper. Res., 11 (1986), pp. 261-294.
- [10] I. KARATZAS, J.P. LEHOCZKY, AND S.E. SHREVE, *Optimal Portfolio and Consumption Decisions for a "Small Investor" on a Finite Horizon*, SIAM J. Control Optim., 25 (1987), pp. 1557-1586.
- [11] I. KARATZAS AND S.E. SHREVE, *Methods of Mathematical Finance*, Springer, New York (1998)

- [12] I. KARATZAS AND H. WANG, *Utility Maximization with Discretionary Stopping*, SIAM J. Control Optim., 39 (2000), pp. 306-329.
- [13] B. O KSENDAL, *Stochastic Differential Equations: An Introduction with Applications*, 5th ed. Springer, New York (1998)
- [14] R.C. MERTON, *Lifetime Portfolio Selection Under Uncertainty: The Continuous-Time Case*, Rev. Econ. Stat., 51 (1969), pp. 247-257.
- [15] R.C. MERTON, *Optimum Consumption and Portfolio Rules in a Continuous-Time Model*, J. Econ. Theory, 3 (1971), pp. 373-413.
- [16] Y.H. SHIN, B.H. LIM, U.J. CHOI, *Optimal Consumption and Portfolio Selection Problem with Downside Consumption Constraints*, Appl. Math. Comput., 188 (2007), pp. 1801-1811.

# A Posteriori Error Analysis of Triangular Mixed Finite Element Methods for Semilinear Quadratic Optimal Control Problems

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In this paper, we present an a posteriori error analysis of semilinear quadratic constrained optimal control problems using triangular mixed finite element methods. The state and co-state are approximated by the order  $k \leq 1$  Raviart-Thomas mixed finite spaces and the control approximated by piecewise constant functions. We derive a posteriori error estimates for the coupled state and control approximations. Numerical test results confirming our theoretical results are also presented.

This is a joint work with Y. P. Chen.

# On the $L^2$ a Priori Error Estimates to the Finite Element Solution of Elliptic Problems with Singular Adjoint Operator

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The Aubin-Nitsche trick for the finite element method of Dirichlet boundary value problem is a well-known technique to obtain a higher order a priori  $L^2$  error estimation than  $H_0^1$  estimates by considering the regularly dual problem. However, as far as the authors concern, when the dual problem is singular, it was not known at all up to now whether the a priori order of  $L^2$  error is still higher than  $H_0^1$  error.

In this paper, we propose a technique for getting a priori  $L^2$  error estimation by some verified numerical computations for the finite element projection. This enables us to obtain the higher order  $L^2$  a priori error than  $H_0^1$  error, even though the associated dual problem is singular. Note that our results are *not a posteriori estimates* but *the determination of a priori constants*.

This is a joint work with Kouji Hashimoto, Mitsuhiro T. Nakao.

# A Simple Proof of the Cucker-Smale Flocking Dynamics and Mean-field Limit

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We present a simple proof on the formation of flocking to the Cucker-Smale system based on the explicit construction of a Lyapunov functional. Our results also provide a unified condition on the initial states in which the exponential convergence to flocking state will occur. For large particle systems, we give a rigorous justification for the mean-field limit from the many particle Cucker-Smale system to the Vlasov equation with flocking dissipation as the number of particles goes to infinity.

# The Uniqueness and Existence of Chern-Simon Equation

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We use the idea of Ricci flow to show the uniqueness of trivial solution to Chern-Simon equation with Dirichlet or Neumann boundary condition. On the other hand, the nonzero boundary condition yields existence of nontrivial solution to the equation. To explore the behavior of the solution, we study both the symmetry solution on disk and solution on star-like domain. Under positive boundary condition, we show that the solution is monotone and constant sign.

# Initial-Boundary Value Problems for Conservation Laws with Source Terms and the Degasperis-Procesi Equation

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In this paper we study the initial boundary value problem for scalar conservation laws with source terms possessing limited regularity. We first define a strong notion of trace at the boundary of  $(0, T) \times \Omega$  reached by  $L^1$  in order to find a good boundary condition and we prove the well-posedness for scalar conservation laws with source terms. The proof is based on the kinetic formulation and the compensated compactness method. Finally, we use those results for the wellposedness of the the initial boundary value problem for the Degasperis-Procesi equation. It is a third order nonlinear dispersive equation that can be rewritten in the form of a conservation with a nonlocal source, and models unidirectional shallow water waves.

This is a joint work with Giuseppe Maria Coclite and Kenneth H. Karlsen.

# Rigorous Error Bounds for Floating-Point Operations without Changing Rounding Mode

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Recently there is growing interest in so-called error-free transformations. They rigorously use properties of floating-point arithmetic. Because only ordinary floating-point operations are used and the rounding mode is not changed, such algorithms are very fast. In this talk we present some new applications, among them the computation of predecessor and successor of a floating-point number. This can be used to simulate interval operations in rounding to nearest without changing the rounding mode. **On the Finite Difference**

## Approximation for the Semilinear Wave Equation

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We consider the semilinear wave equation  $u_{tt} = u_{xx} + u^2$  ( $0 < x < 1$ ,  $t > 0$ ) and its finite difference analogue, whose solutions blow up in finite time. We report our recent results on the convergence and the asymptotic behaviour of the finite difference solution.

This is a joint work with H. Okamoto (RIMS, Kyoto U., Japan)

# SWE Model Based Simulation to Analyze the Cause of Pohang Harbor Hazard

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The Pohang harbor located at northeast part of Pohang City has been experienced extreme wave hazard of about 3.0-5.00 meters high, and more than 7 meters sometimes. To resolve this problem, high performance numerical simulations, which are based on Shallow Water Equation(SWE), are conducted to investigate the security of the harbor and then to propose tactics of improvements. Numerical evaluation scheme is composed of three parts i.e., bathymetry data, wave propagation simulation and scenarios of wave source. High resolution topography and bathymetry data are used near the harbor area to realize the actual situation precisely. Wave propagation on the topography is simulated using SWE with the near-field tsunami propagation phenomena. Then a staggered leap-frog scheme in finite difference method has been used to solve this nonlinear problem. Animations made from simulation results exhibit reflection, defraction and interference of the wave propagation. Further, applying Power Spectrum Density (PSD) analysis to the time series data, we obtained resonance frequencies at the track recorded points in the harbor.

This is a joint work with Kwang Ik Kim (Pohang Univ. of Sci. & Tech), Huai Zhang (Graduate University of Chinese Academy of Sciences), Yaolin Shi (Graduate University of Chinese Academy of Sciences).

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# Verified Computations of Bifurcating Solutions for 3-dimensional Rayleigh-Bénard Convection Problems

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In this talk, we present a three dimensional extension of the two dimensional works on a computer assisted proof of the existence of nontrivial steady state solutions for Rayleigh-Bénard convection based on the fixed point theorem using a Newton like operator. The differences are emerging of complicated types of bifurcation, direct attack on the problem without stream functions, and increased complexity of numerical computation. The last one makes it hard to proceed the verification of solutions corresponding to the points on bifurcation diagram for three dimensional case. Actually, this work should be the first result for the three dimensional Navier-Stokes problems which seems to be very difficult to solve by theoretical approaches.

This is a joint work with Mitsuhiro T. Nakao, Yoshitaka Watanabe and Takaaki Nishida.

# New Directions of Statistical Physics in the Age of Computational Science: Yang-Lee Zero Phase Transition Approach for 2-D Triangular Antiferromagnets

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After the advent of computers, the way of doing science and engineering research has been changed dramatically. Computational sciences has been added to the traditional pillars of science and engineering; theory and experiment or observation. Along this way, in statistical physics a new mechanism for the occurrence of phase transitions, so called Yang-Lee zero approach, has been proposed and developed. In this presentation, I will give a simple example; Yang-Lee zeros of the triangular Ising antiferromagnets.

JointWork: with Seung-Yeon Kim at Chungju National University.

# Counting Closed Orbits of Hyperbolic Diffeomorphisms via Mertens Theorem

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In this work we consider two types of extensions of a hyperbolic diffeomorphism restricted to a basic set, namely, a finite group extension and an automorphism extension. In particular we are interested in the way closed orbits of the base transformation lifts onto the extension spaces. Moreover using the counting function motivated by Mertens theorem of classical number theory, we provide asymptotic estimates for the number of closed orbits of the hyperbolic diffeomorphism according to the lifting patterns.

# New Singular Function Method for Domain Singularities

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## Abstract

Recently, a new singular function(NSF) method was posed to get accurate numerical solution on quasi-uniform grids for two-dimensional Poisson and interface problems with domain singularities. Using the singular function representation of the solution, dual singular functions, and an extraction formula for stress intensity factors, the method poses a weak problem whose solution is in  $H^2(\Omega)$  or  $H^2(\Omega_i)$ . In this paper, we show that the singular functions, which are not in  $H^2(\Omega)$ , also satisfy the integration by parts and note that this fact suggests the possibility of different choice of the weak formulations. We show that the original choice of weak formulation of NSF method is critical.

## References

- [1] I. BABUSKA, R.B. KELLOGG, AND J. PITKARANTA, *Direct and inverse error estimates for finite elements with mesh refinements*, Numer. Math., 33 (1979), 447-471.
- [2] H. BLUM AND M. DOBROWOLSKI, *On finite element methods for elliptic equations on domains with corners*, Computing, 28 (1982), 53-63.
- [3] M. BOURLARD, M. DAUGE, M.-S. LUBUMA, AND S. NICAISE, *Coefficients of the singularities for elliptic boundary value problems on domains with conical points III. Finite element methods on polygonal domains*, SIAM Numer. Anal., 29 (1992), 136-155.
- [4] S. C. BRENNER, *Multigrid methods for the computation of singular solutions and stress intensity factor I: Corner singularities*, Math. Comp., 68 (226), (1999), 559-583.
- [5] S. C. BRENNER AND L.-Y. SUNG, *Multigrid methods for the computation of singular solutions and stress intensity factors III: Interface singularities*, Comput. Methods Appl. Mech. Engrg. 192(2003), 4687-4702.
- [6] Z. CAI AND S.C. KIM, *A finite element method using singular functions for the Poisson equation: Corner singularities*, SIAM J. Numer. Anal., 39:(2001), 286-299.
- [7] Z. CAI, S.C. KIM, S.D. KIM AND S. KONG, *A finite element method using singular functions for the Poisson equation: Mixed boundary condition*, Computer Methods in Applied Mechanics and Engineering, 195:(2006), 2635-2648.
- [8] Z. CAI, S. KIM, AND B.-C. SHIN, *Solution methods for the Poisson equation: Corner singularities*, SIAM J. Sci. Comput., SIAM J. SCI. COMPUT., 23:(2001), 672-682.
- [9] M. DJAOUA, *Equations Intégrales pour un Probleme Singulier dans le Plan*, These de Troisieme Cycle, Universite Pierre et Marie Curie, Paris, 1977.
- [10] M. DOBROWOLSKI, *Numerical Approximation of Elliptic Interface and Corner Problems*, Habilitationsschrift, Bonn, 1981.
- [11] G. J. FIX, S. GULATI, AND G. I. WAKOFF, *On the use of singular functions with finite elements approximations*, J. Comput. Phys., 13 (1973), 209-228.
- [12] V. GIRAULT AND P. A. RAVIART, *Finite element methods for Navier-Stokes equations : theory and algorithms* , Springer-Verlag,Berlin, 1986.
- [13] P. GRISVARD, *Elliptic Problems in Nonsmooth Domains*, Pitman, Boston, MA, 1985.
- [14] R.B. KELLOGG, *Singularities in interface problems*, in: B. Hubbard(ED.), *Numerical Solution of Partial Differential Equations II*, Academic Press, New York, (1971) 351-400.

- [15] R.B. KELLOGG, *On the Poisson equation with intersecting interfaces*, Appl. Anal. 4(1975) 101-129.
- [16] S.C. KIM, Z. CAI, J.H. PYO AND S. KONG, *A finite element method using singular functions: interface problems*, Hokkaido Mathematical Journal, To appear.
- [17] D. MERCIER, *Minimal regularity of the solutions of some transmission problems*, Technical Report 01.7, Universite de Valenciennes et du Hainaut-Cambresis, 2001.
- [18] S. NICAISE, *Polygonal Interface Problems*, Peter Lang, Frankfurt am Main, 1993.
- [19] A. SCHATZ AND L. WAHLBIN, *Maximum norm estimates in the finite element method on plane polygonal domains*, Part 1, Math. Comp., 32 (141), (1978) 73-109; Part 2 (refinements), Math. Comp., 33 (146), (1979) 465-492.
- [20] CH. SCHWAB, *p- and hp-Finite Element Methods*, Oxford University Press, Oxford, 1998.
- [21] B. A. SZABÓ AND I. BABUSKA, *Finite Element Analysis*, John Wiley & Sons, New York, 1991.

# A Set of Variant Hermite Elements for Dirichlet Boundary Problems and its Applications

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An accurate scientific computing requires a large memory capacity and takes a long CPU time. It is important to pay attention to increasing costs for accurate approximations. We employ finite elements of polynomials of degree three to solve PDEs more efficiently. The Lagrange and Hermite elements are well-known as  $H^1$ -conforming ones. The former has only degrees of freedom (DOF) of function values. On the other hand the latter has also ones of derivative values. Hence the code needs to be programmed on a case-by-case basis in dealing with the Dirichlet boundary conditions. However the Hermite element is remarkable in that the total number of DOF is much smaller than that of the Lagrange one. In this paper we present a set of variant Hermite elements where the derivative value degrees on the boundary are replaced by some function value ones. This can deal with the Dirichlet boundary conditions easily because they are determined by only function value degrees. These elements are also  $H^1$ -conforming and the number of DOF is as small as the conventional Hermite element. To obtain the expected convergence order for general problems with curved boundaries we use the isoparametric scheme to the variant Hermite element, where the mapping is equivalent to the Lagrange one. A priori error estimate is verified for the Poisson problem. Some numerical examples are shown for the 2- and 3-dimensional Poisson and Stokes problems. We also observe the effectiveness of the isoparametric method in comparison with the affine scheme. The merit of the variant Hermite element scheme consists in the simplicity of the program and the memory reduction for computations.

JointWork: (adviser:M. Tabata(Kyushu University))